可逆プログラミング言語

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Example: Fibonacci-Pairs

```
n x1 x2
```

global store (three integer variables, initially zero)

```
procedure fib
  if n=0 then x1 += 1
       test  x2 += 1
         else n -= 1
              call fib
              x1 += x2
              x1 <=> x2
  fi x1=x2
       assertion
```

reversible procedure

swap values of x1, x2

Forward & Backward Computation

```
procedure main_fwd
  n += 4
  call fib
```

procedure forward

```
procedure main_bwd
  x1 += 5
  x2 += 8
  uncall fib
```

procedure backward

```
procedure fib
```

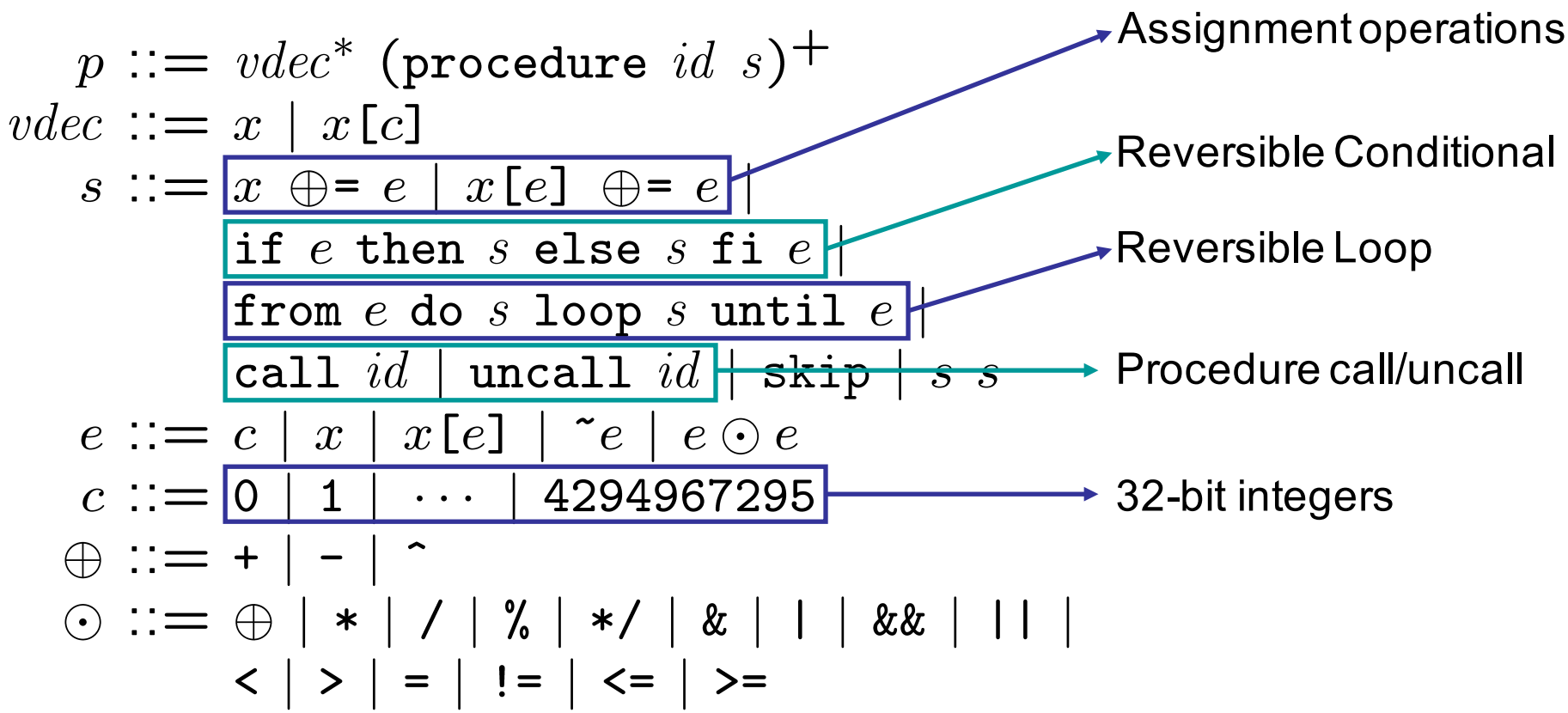
same procedure!
(code sharing)

[ReillyFederighi65,LutzDerby82]

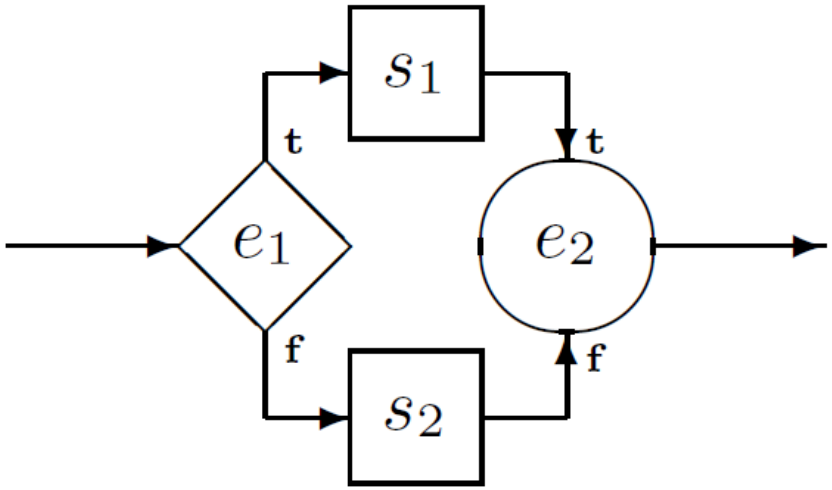
Janus: a Reversible Language

- To our knowledge, the first reversible structured language
  - Suggested for a class at Caltech [Lutz and Dervy 1982]
- Imperative language
- Global store, no local store
- Scalar and array types, integer values
- Structured control operators (IF, LOOP)
- Simple procedures (correspond to loops)
  - No return value, side effects on global store

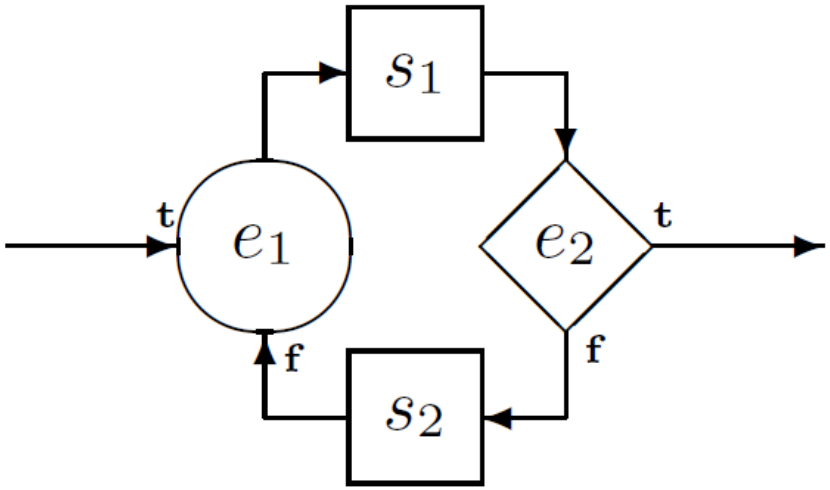
Syntax of Janus



Control Flow Operators



(a) Conditional



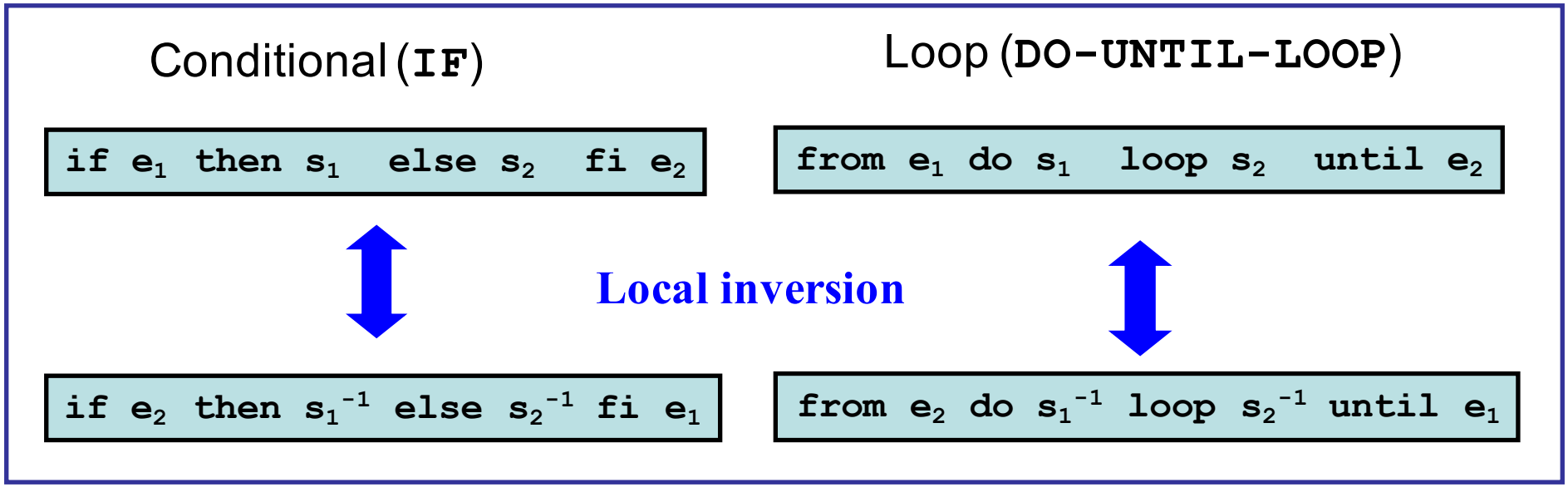
(b) Loop

`if e1 then s1 else s2 fi e2`

`from e1 do s1 loop s2 until e2`

Remark: Circles are assertions

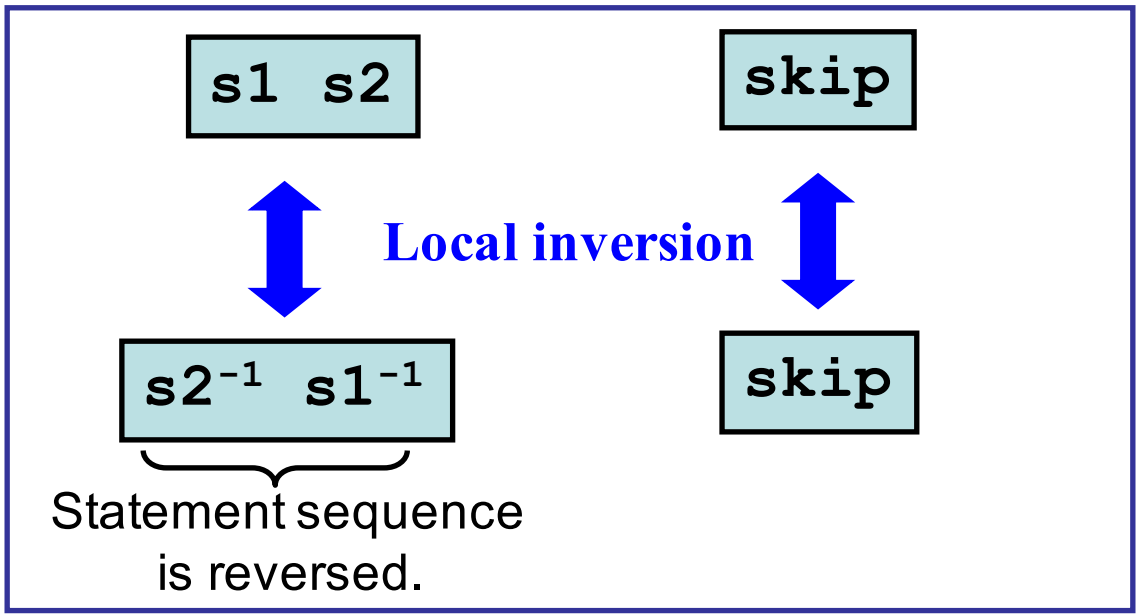
Local Inversion of CFOs



$\Rightarrow$ Conditional and Loop are reversible.

Skip and Sequence

$$\frac{\sigma \vdash_{stmt} s_1 \Rightarrow \sigma' \quad \sigma' \vdash_{stmt} s_2 \Rightarrow \sigma''}{\sigma \vdash_{stmt} s_1\ s_2 \Rightarrow \sigma''}\ \text{Seq} \qquad \frac{}{\sigma \vdash_{stmt} \texttt{skip} \Rightarrow \sigma}\ \text{Skip}$$



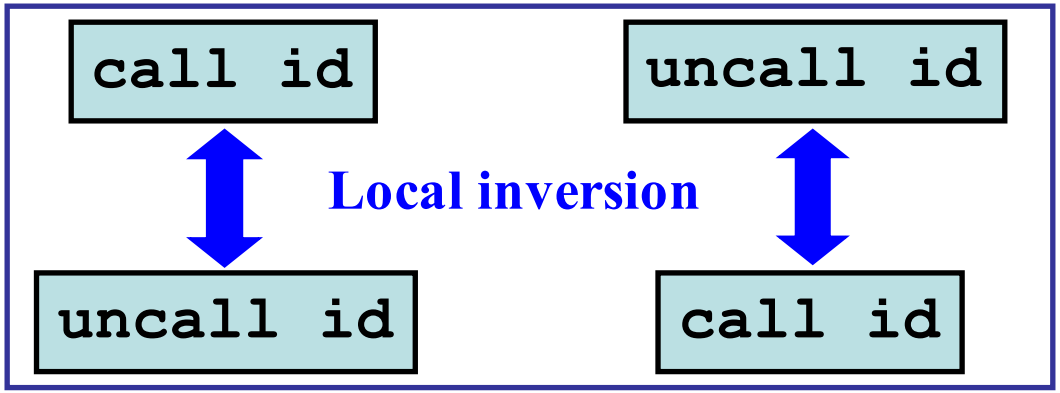
⇒ Skip and sequence are reversible.

Procedure Call / Uncall

$$\frac{\boxed{\sigma} \vdash_{stmt} \Gamma(id) \Rightarrow \boxed{\sigma'}}{\sigma \vdash_{stmt} \texttt{call}\ id \Rightarrow \sigma'}\ \text{Call}$$

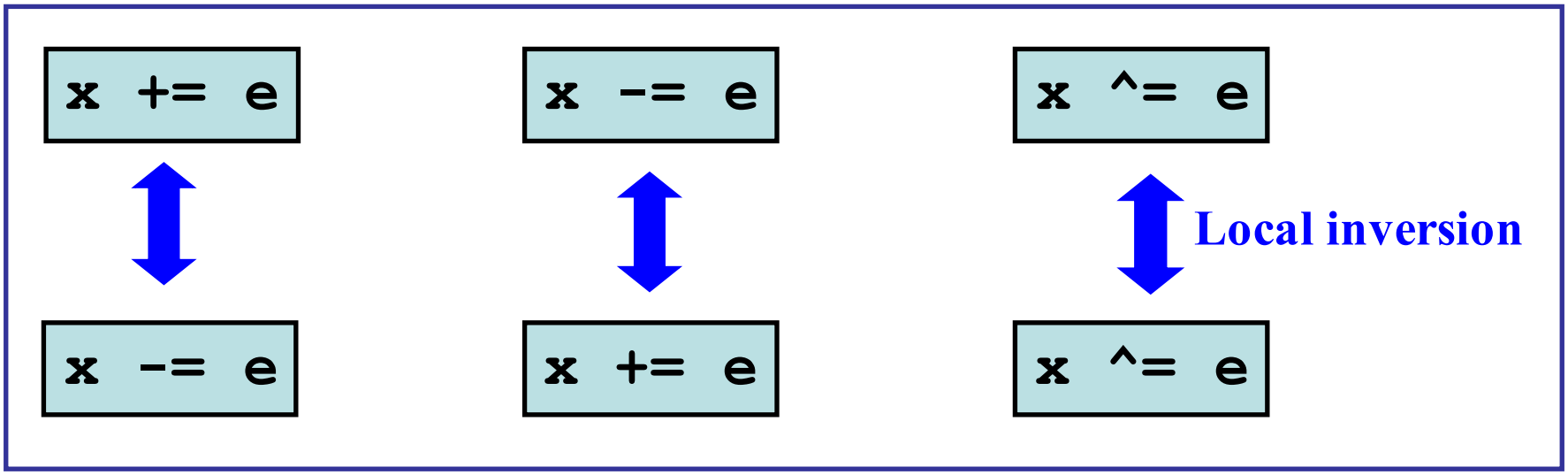
$$\frac{\boxed{\sigma'} \vdash_{stmt} \Gamma(id) \Rightarrow \boxed{\sigma}}{\sigma \vdash_{stmt} \texttt{uncall}\ id \Rightarrow \sigma'}\ \text{Uncall}$$

$\Gamma \in$ Idens[Janus] $\rightharpoonup$ Stmts[Janus]



$\Rightarrow$ Procedure call / uncall is reversible.

C-like Assignments



Abbreviation: `x += e` ⇔ `x := x + e`
If variable `x` must not occur in expression `e`,
this is again an example of reversible update.

⇒ C-like Assignments are reversible.

Evaluation of Expressions

Judgment: $\sigma \vdash_{expr} e \Rightarrow v$

Store Exp Val

$$\frac{}{\sigma \vdash_{expr} c \Rightarrow [\![c]\!]}\ \text{Con} \qquad \frac{}{\sigma \vdash_{expr} x \Rightarrow \sigma(x)}\ \text{Var}$$

$$\frac{\sigma \vdash_{expr} e_1 \Rightarrow v_1 \quad \sigma \vdash_{expr} e_2 \Rightarrow v_2 \quad [\![\odot]\!](v_1, v_2) = v}{\sigma \vdash_{expr} e_1 \odot e_2 \Rightarrow v}\ \text{BinOp}$$

$$\odot \in \{+, -, \hat{}, \ldots\}$$

Store σ : Var $\Rightarrow$ Val

Evaluation of expressions is fwd deterministic.

But it is not backward deterministic.

Non-injective Binary Operators

- Some of the binary operators (others are similar)

$$[\![+]\!](v_1, v_2) = (v_1 + v_2) \mod 2^{32}$$

$$[\![=]\!](v_1, v_2) = \begin{cases} 0 \ if \ v_1 \neq v_2 \\ 1 \ if \ v_1 = v_2 \end{cases}$$

- No binary operator is injective.
- There does not exist a unique inverse operation.

Question: Why does this *not* harm the reversibility of statements?

Answer: Reversible Update

Evaluation of RHS (Irreversible)

Reversible Update

$$\frac{\sigma \vdash_{expr} e \Rightarrow v \qquad v_2 = [\![\oplus]\!](v_1, v)}{\sigma \uplus \{x \mapsto v_1\} \vdash_{stmt} x \oplus= e \Rightarrow \sigma \uplus \{x \mapsto v_2\}}\ \text{AssVar}$$

- $\sigma \vdash_{expr} e \Rightarrow v$ is fwd deterministic.
- Variable x must not occur in expression e.
- Function $\lambda v'.\ [\![\oplus]\!](v', v)$ is injective for any v

when $\oplus$ is + , - or ^ .

$\Rightarrow$ It has an inverse function.

C-like Assignments are reversible.

Theorem: Janus Statements are Reversible

$$\forall s \in \text{Stmts[Janus]}, \exists s' \in \text{Stmts[Janus]},$$
$$\forall \sigma, \sigma' \in \text{Stores[Janus]}.$$
$$\sigma \vdash_{stmt} s \Rightarrow \sigma' \iff \sigma' \vdash_{stmt} s' \Rightarrow \sigma$$

Remarks:

- Evaluation of expressions is not reversible.
  But this does not harm this reversibility.
- Referential transparency: $s = s' \Rightarrow s_1\ s\ s_2 = s_1\ s'\ s_2$
- We cannot write irreversible programs in Janus.

Criteria of Computational Strength

R-Turing completeness

A reversible language is called r-Turing complete

if it can simulate reversible Turing machines (RTM), cleanly.

RTM in Janus:
Less than 40 lines

```
procedure main
 ... RTM, tape and constants decl. and init.
 from  q=QS
 do    call inst(q,left,s,right,q1,s1,s2,q2,
       pc += 1
       if pc=PC_MAX then
          pc ^= PC_MAX
       fi pc=0
 until q=QF

procedure pushtape
 if empty(stk) && (s=BLANK) then
    s ^= BLANK   // zero-clear s
 else
    push(s,stk)
 fi empty(stk)
```

```
procedure inst

 if q=q1[pc] then
   if s=s1[pc] then                  // Symbol rule:
     q += q2[pc]-q1[pc]              // set q to q2[pc]
     s += s2[pc]-s1[pc]              // set s to s2[pc]
   else
     if s1[pc]=SLASH then            // Shift rule:
       q += q2[pc]-q1[pc]            // set q to q2[pc]
       if s2[pc]=RIGHT then
         call   pushtape(s,left)     // push s on left
         uncall pushtape(s,right)    // pop right to s
       else
         if s2[pc]=LEFT then
           call   pushtape(s,right)  // push s on right
           uncall pushtape(s,left)   // pop left to s
         fi s2[pc]=LEFT
       fi s2[pc]=RIGHT
     fi s1[pc]=SLASH
   fi s=s2[pc]
 fi q=q2[pc]
```

- Assignment:
  - Zero-cleared copying, Zero-clearing by a constant

```
{ x := 0, y := v }
x ^= y
{ x := v, y := v }
```

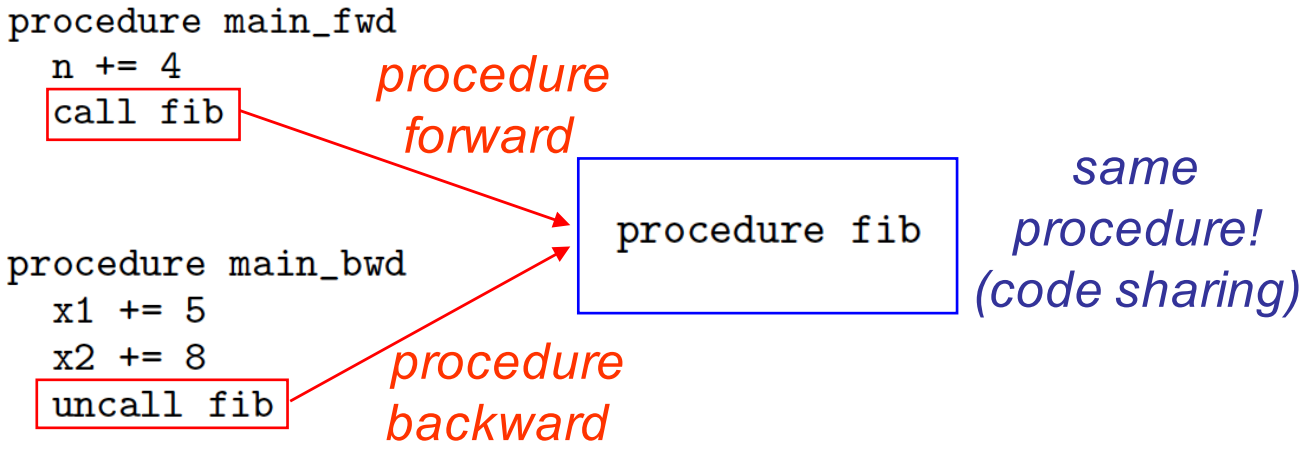
```
{ x := v, y := v }
x ^= y
{ x := 0, y := v }
```

- Garbage manipulation:
  - Temporary stack

```
procedure alloc_tmp
   tmp_sp += 1
   tmp <=> tmp_stack[tmp_sp]
```

```
{ x := v, ... }
x <=> tmp
call alloc_tmp
{ x := 0, ... }
```

- New modularity:
  - Code sharing by call and uncall

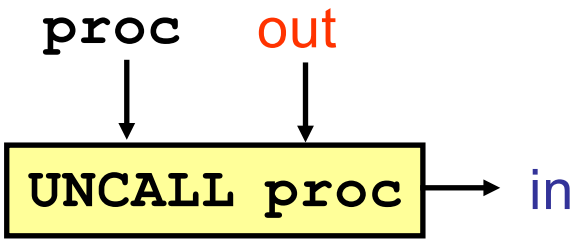


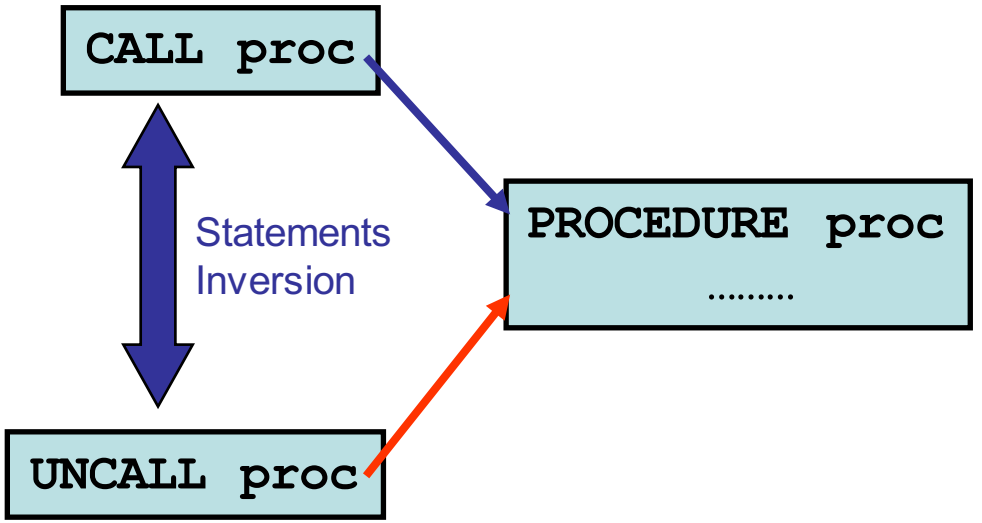
- Call-uncall (Garbage collection)
  - Local Bennett's method [Bennett 1973]:

```
call f
// copy the result of f
uncall f
```

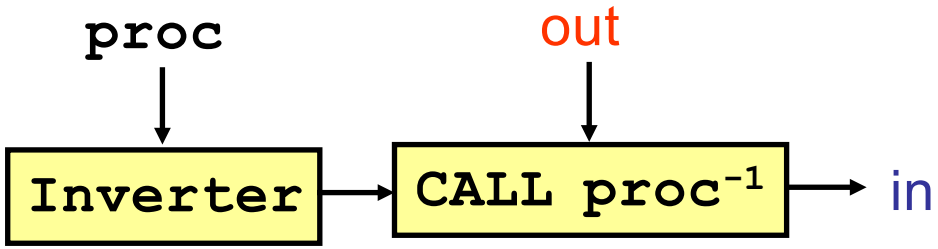
Two Approaches to Inversion of Program

- Inverse Interpretation:

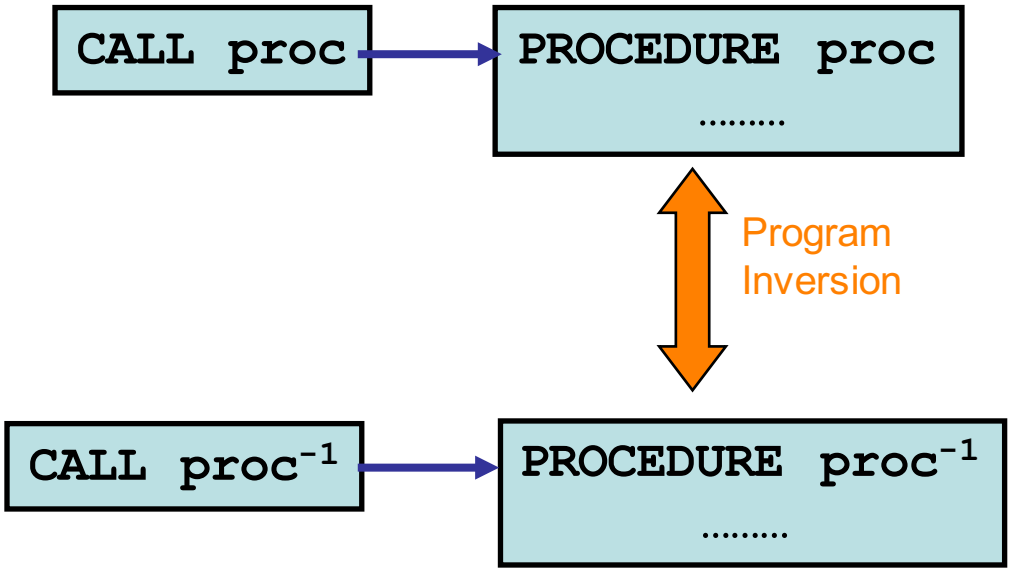




- Program Inversion:



In Janus, any statements have its inverse.



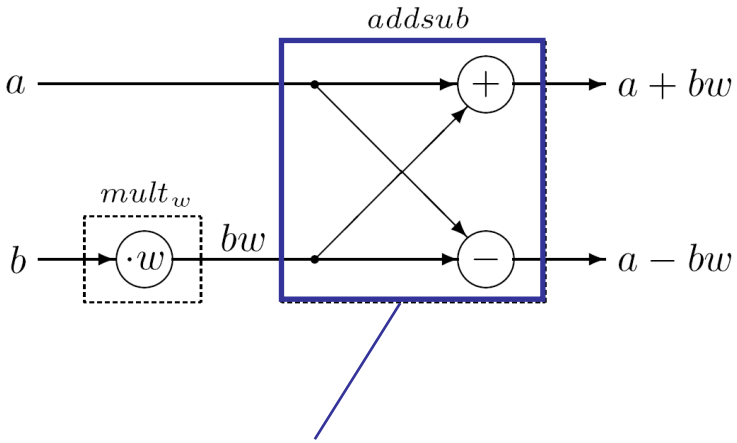
Reversible Integer FFT (radix-2) [CF08]

```
procedure rfft(int re, int im, int N)
  local int k=0, int j=0, int m=1
    from m=1                    // Stage of butterflies
    do from k=0
       do from j=0                                          // Butterfly:
          do re[k+j+m] += lift1(k,j,m,im)                   // 1. lifting
             im[k+j+m] += lift2(k,j,m,re)
             re[k+j+m] += lift1(k,j,m,im)
             call flipswap(k,j,m,re,im)
             +-=(re[k+j],re[k+j+m])                         // 2. addsub
             +-=(im[k+j],im[k+j+m])
             j += 1
          until j=m
          j ^= m
          k += m*2
       until k=N
       k ^= N
       call double(m)
    until m=N
  delocal int k=0, int j=0, int m=N
```

Computational Kernel

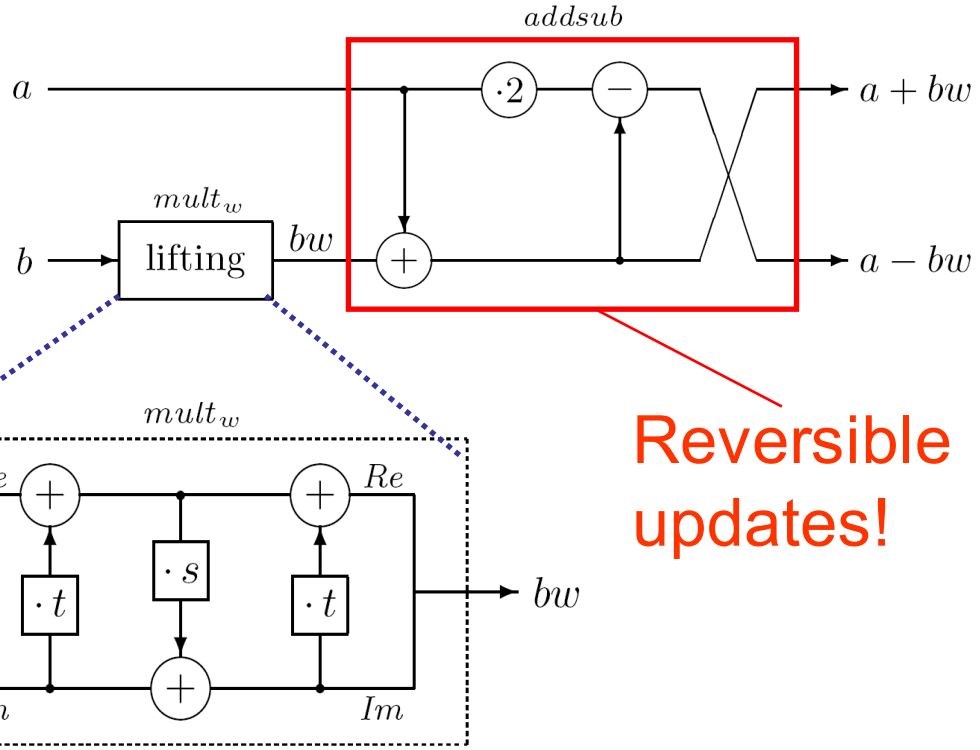
Computational Kernel

Ordinary butterfly



Add and subtract real and imaginary parts of a and b.

Reversible butterfly



Reversible updates!

Lifting

[OraintaraChenNguyen02]

Concluding Remarks

- As any computation model does, reversible computation model itself is theoretically of interest.

- Formalized reversible language Janus.
  - Janus: the first reversible language suggested for a class at Caltech [Lutz 1986].
- Proved that Janus is reversible.
- Explored the connection between program inversion and reversible computing.
- Demonstrated the practical and nontrivial reversible programs
  - fast Fourier transform
- Shown the computational strength of the language by implementing a reversible Turing machine interpreter.

Related Work: History of (Clean) Reversible High-level Programming Languages

- Janus [Lutz and Derby 1982]
  - The *first* reversible language. Imperative.
- psiLisp [Baker 1992]
  - The reversible Lisp-like functional language w/destructive updates.
- R [Frank 1997]
  - R compiler generates PISA code, which *runs on the reversible processor* Pendulum [Vieri 1999].
- Inv [Mu, Hu, and Takeichi 2004]
  - An injective *functional* language.
- Gries' invertible language [Gries 1981]
  - Locally invertible CFOs

References

- Yokoyama T., Glück R., A reversible programming language and its invertible self-interpreter. In: Partial Evaluation and Program Manipulation. Proceedings. 144-153, ACM Press 2007.
- Axelsen H.B., Glück R., Yokoyama T., Reversible machine code and its abstract processor architecture. In: Diekert V., et al. (eds.), Computer Science - Theory and Applications. Lecture Notes in Computer Science, Vol. 4649, 56-69, Springer-Verlag 2007.
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